Rejoinder

Huang Huang, Sameh Abdulah, Ying Sun, Hatem Ltaief,
David E. Keyes, and Marc G. Genton

Firstly, we are grateful to the discussants for providing details of their methods used in the *Competition on Spatial Statistics for Large Datasets*, including the teams AppStatUZH, Colorado-School-of-Mines, GpGp, RESSTE, Tohoku University, and UOW.

A summary of the competition and main contributions has been emphasized by several discussants. Through the competition and discussions provided, we have better understood when each approximation method became inadequate. We have also released a large set of data generated by various spatial processes with different parameter settings together with the lowest RMSEs achieved by researchers worldwide.

With the valuable insights from the discussants, we highlight several findings from this competition for spatial process inferences or predictions as follows:

- Accurate estimation of the smoothness parameter is generally a difficult problem. Among the parameter settings we have considered, we observe that the estimation of the smoothness is more challenging for smoother and noisier processes. As pointed out by the team RESSTE, when the smoothness is high, which leads to the regularity of the covariance to certain extents, and the nugget is present, estimates of the smoothness become comparatively poorer. However, in this competition, the datasets were large, and there were enough pairs of observations at small distances so that the estimation of the smoothness did not appear to be an issue among well-performing methods, as noticed by the team GPGP.
- Given the large datasets, estimates of nuggets are always very accurate among well-performing methods.
- As emphasized by the team Colorado-School-of-Mines, when approximation methods are used, a better parameter estimation does not always guarantee that the prediction is more accurate because the approximation induced in the prediction also plays a critical role.

H. Huang · S. Abdulah · Y. Sun · H. Ltaief · D. E. Keyes · M. G. Genton (✉)
King Abdullah University of Science and Technology (KAUST), Thuwal 23955-6900, Saudi Arabia
(E-mail: *marc.genton@kaust.edu.sa*).

Journal of Agricultural, Biological, and Environmental Statistics, Volume 26, Number 4, Pages 621–623
https://doi.org/10.1007/s13253-021-00471-1

- As pointed out by the team AppStatUZH, the RMSE is significantly larger in the presence of the nugget due to a lower signal-to-noise ratio. In addition, when the smoothness is smaller, the prediction is more challenging and tends to have larger errors.

There still exists abundant room for improvement that can direct future competition organization:

- Datasets. The team UOW points out that the spatial processes we have considered in this competition are still limited and lack cases that may appear in practice. The team Colorado-School-of-Mines raises the fact that general good predictions may be due to the uniformly spaced locations that ensure enough training neighbors for every testing location, and the team GpGp suggests a data location pattern mimicking a polar orbiting satellite. The team RESSTE also discusses more challenging non-Gaussian data. We agree with these comments and suggestions. This competition only considered a fundamental setting to generate spatial datasets. Future competitions could definitely design more realistic simulation schemes representing important geostatistical problems. In fact, we plan to organize more competitions with an emphasis on nonstationarity modeling, extension to multivariate random fields and spatiotemporal random fields, big gap-filling problems, to name a few.
- The software *ExaGeoStat*. In this competition, *ExaGeoStat* (Abdulah et al. 2018), available at https://github.com/ecrc/exageostat under the BSD 3-Clause License, was used to generate the datasets and acted as a benchmark. The team UOW praises it for its ability in spatial and spatiotemporal modeling. They also comment that the use of *ExaGeoStat* requires considerable high-performance computing (HPC) expertise. To enhance its user productivity, we also have released an R (R Core Team 2019) package *ExaGeoStatR* (Abdulah et al. 2019) for statisticians who lack HPC expertise. An *ExaGeoStatR* Docker image is available at https://hub.docker.com/r/ecrc/exageostat-r, and the source code is available at https://github.com/ecrc/exageostatr under the BSD 3-Clause License. We are regularly improving and updating *ExaGeoStat* and *ExaGeoStatR* to increase their functionality and user-friendliness.
- Competition assessment. The team UOW brings the issue of uncertainty quantification. In this competition, we did not evaluate the uncertainty of parameter estimation. Instead, in sub-competition 1a, the metrics mean loss of efficiency (MLOE) and mean misspecification of the mean square error (MMOM) (Hong et al. 2021) assess the prediction efficiency where the uncertainty of the random process is considered. Ideally, the uncertainty of parameter estimation could be assessed via replicated data realizations, which may not be practical in a competition. For predictions, in sub-competitions 1b, 2a, and 2b, we only considered point predictions and used RMSE for assessment. It would be possible to evaluate the probabilistic predictions using proper scoring rules, say continuous rank probability score (CRPS, Gneiting and Raftery 2007). However, not all methods can give probabilistic predictions, so we only required point predictions for these three sub-competitions. It is true that point predictions may not differ substantially for nonstationary or non-Gaussian random fields with large datasets.

For future competitions emphasizing predictions of this type, predictive distribution assessment should be well accounted for.

- Computing time. The teams Colorado-School-of-Mines and UOW point out the difficulty for a fair comparison of computing time. One way to resolve this problem is to rerun all the code on our side on the same HPC facility. However, this would not be trivial and become very labor demanding when many submissions are involved. Future competitions may adopt certain intelligent ways of providing a common test bed for computing if the comparison of computational performance is desired.

We remark a typo in the paper *Competition on Spatial Statistics for Large Datasets* (Huang et al. 2021). The x-axis ticks "G1–G16" in the top panel of Figure 3 should be shifted by one spot to the right.

[Received August 2021. Revised August 2021. Accepted August 2021. Published Online August 2021.]

REFERENCES

Abdulah S, Li Y, Cao J, Ltaief H, Keyes DE, Genton MG, Sun Y (2019) ExaGeoStatR: a package for large-scale geostatistics in R. arXiv preprint arXiv:1908.06936

Abdulah S, Ltaief H, Sun Y, Genton MG, Keyes DE (2018) ExaGeoStat: a high performance unified software for geostatistics on manycore systems. IEEE Trans Parallel Distrib Syst 29(12):2771–2784

Gneiting T, Raftery AE (2007) Strictly proper scoring rules, prediction, and estimation. J Am Stat Assoc 102(477):359–378

Hong Y, Abdulah S, Genton MG, Sun Y (2021) Efficiency assessment of approximated spatial predictions for large datasets. Spat Stat 43:100517

Huang H, Abdulah S, Sun Y, Ltaief H, Keyes DE, Genton MG (2021) Competition on spatial statistics for large datasets. J Agric Biol Environ Stat (in press)

R Core Team (2019) R: a language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org/